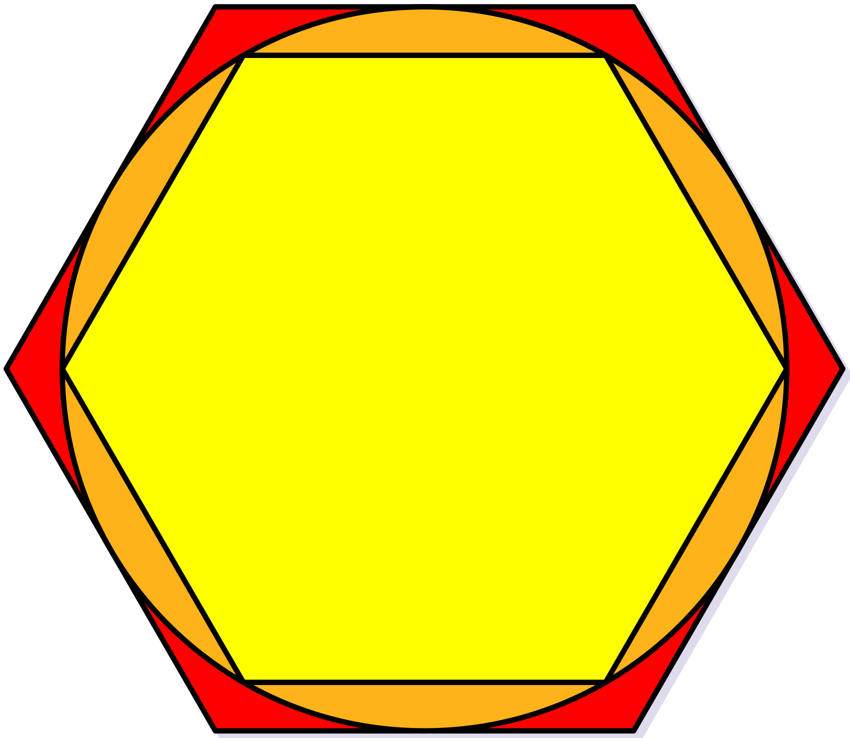
Print ‘n’ Play Collection

Of the 12 Geometrical Puzzles

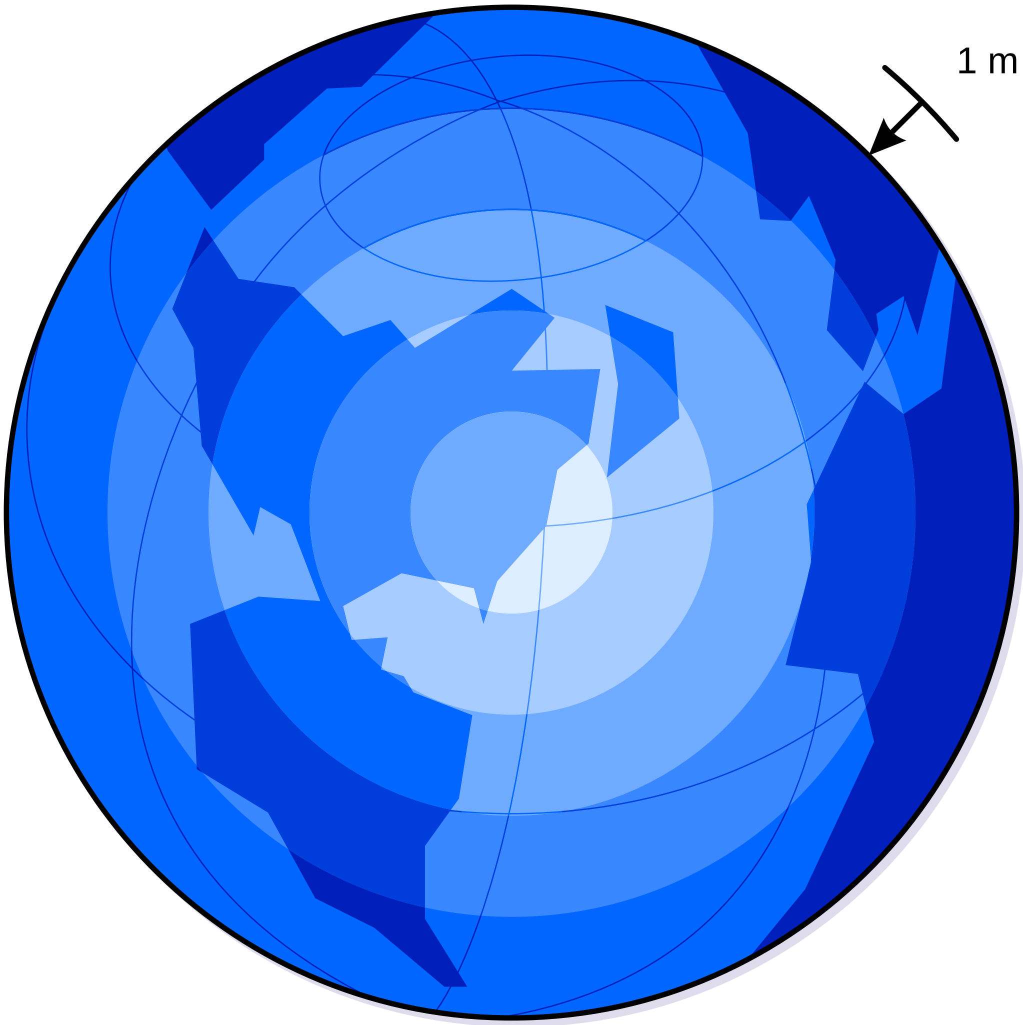
GEOMETRICAL 2 Puzzles

Puzzles.COM



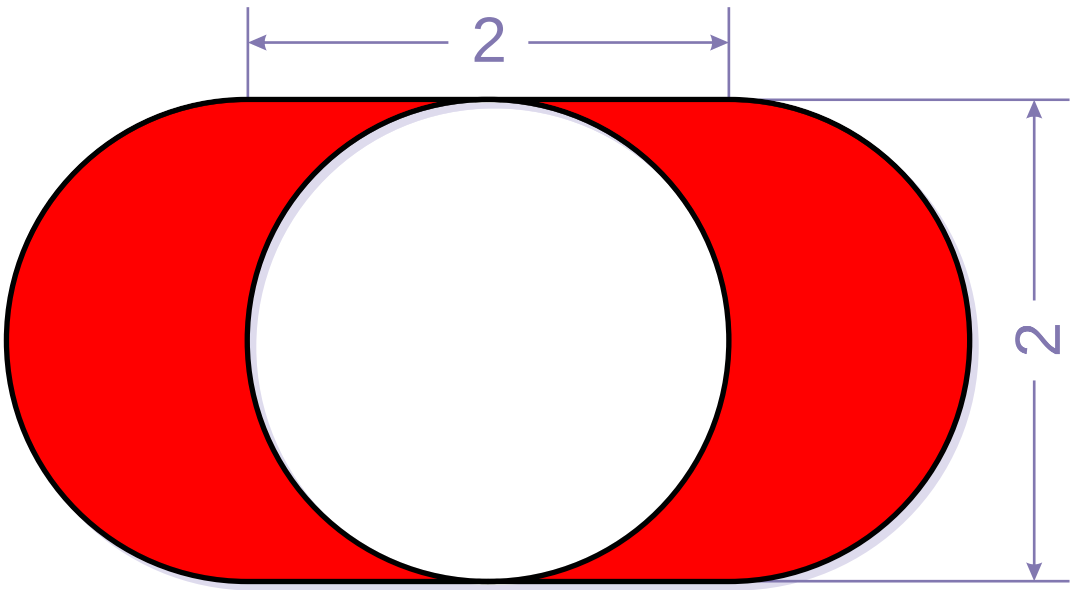
Regular hexagons are inscribed in and circumscribed outside a circle - as shown in the illustration.

If the smaller hexagon has an area of three square units, what is the area of the larger hexagon?

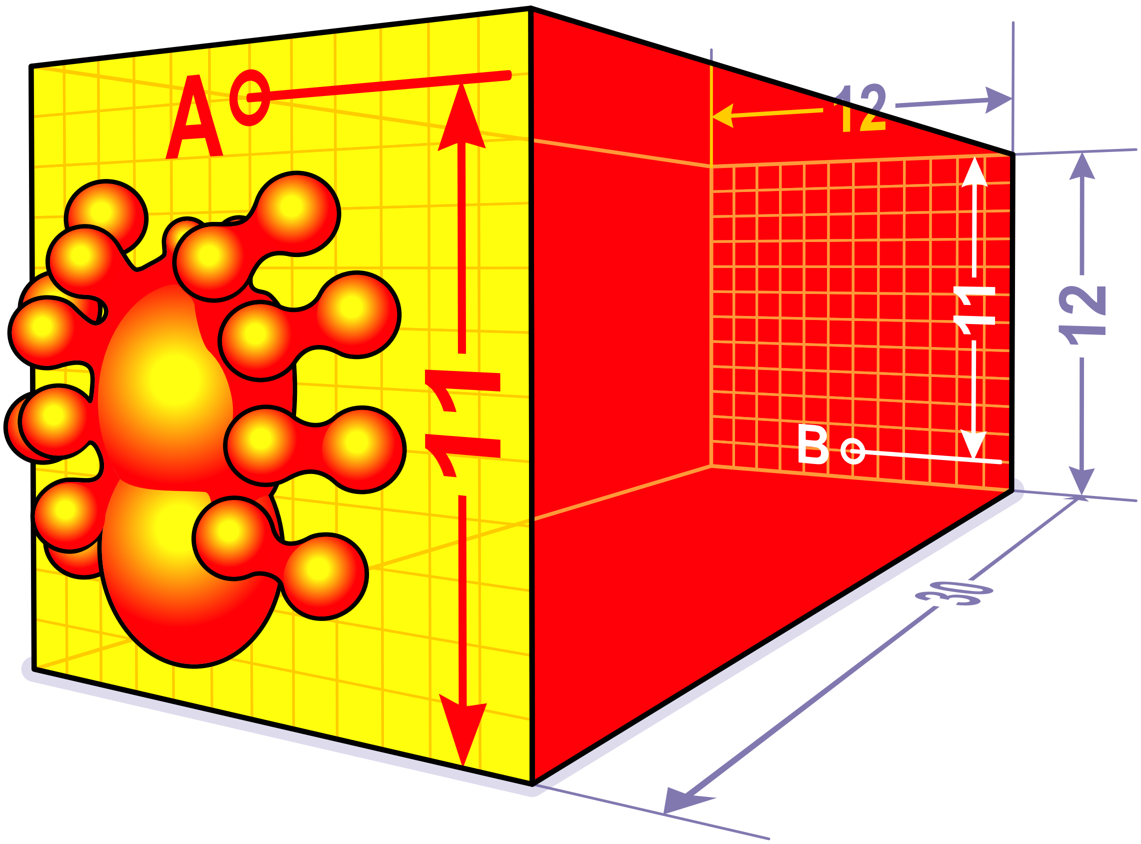


Imagine that you are on a perfectly smooth sphere as big as the earth. A steel belt is stretched tightly around one of its equators.

One meter of steel (a little bit more than a yard) is added to this belt so that it is raised off the surface of the sphere by the same distance all the way around. Will this lift the belt high enough so that you can:
1) Slip a playing card under it?
2) Slip your hand under it?
3) Slip a baseball under it?



A paper sheet has the shape of a two-unit square with semicircles on opposite sides. If a disk with a diameter of two units is removed from the center as shown in the illustration, what is the area of the remaining paper?

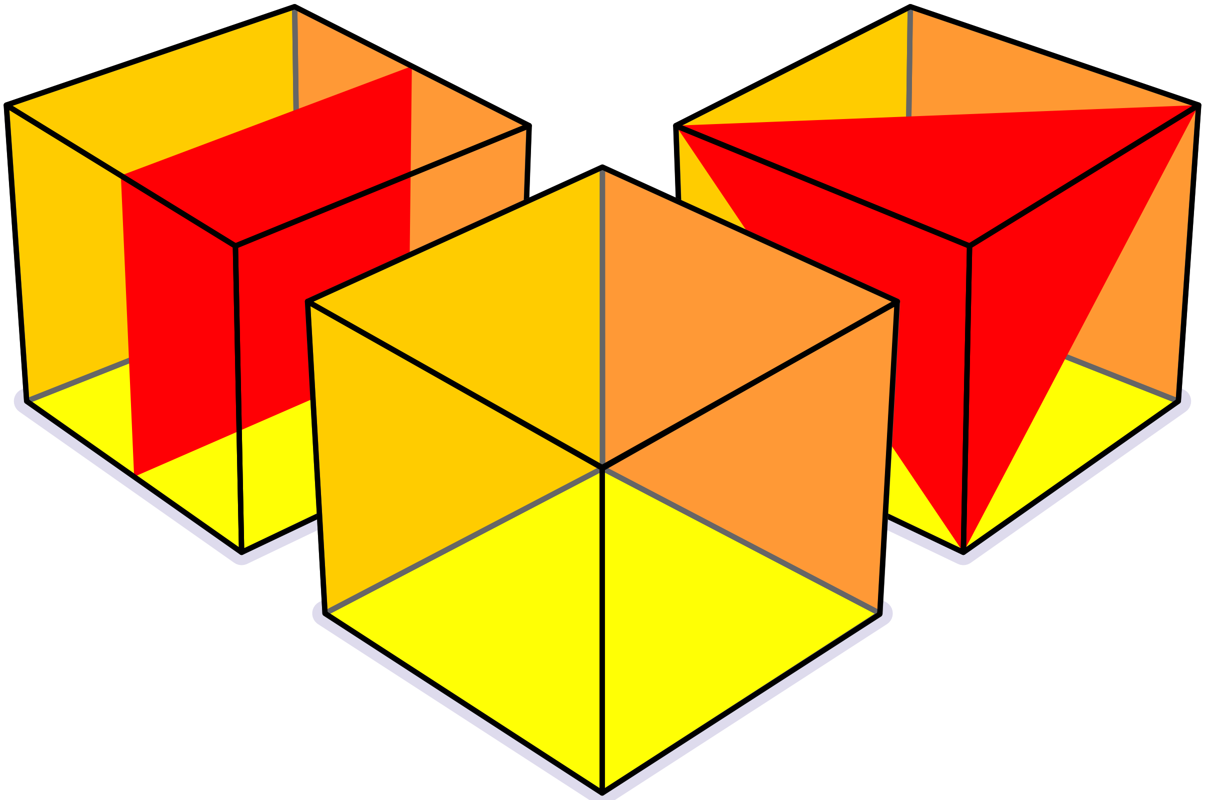


A rectangular box is 30 units long, 12 units wide, and 12 units high. A Spider starting from spot A should reach spot B. Spot A is midway from the sides of the box and 1 unit from its top. Spot B is midway from the sides of the box, 1 unit from the bottom of the box, and on the opposite side from A. The box, its dimensions and the proper locations of spots A and B are shown in the illustration.

What is the shortest way for the Spider to reach spot B? During the journey from A to B the Spider can use any side and edge of the box. Hint: the shortest distance between A and B is less than 42 units.

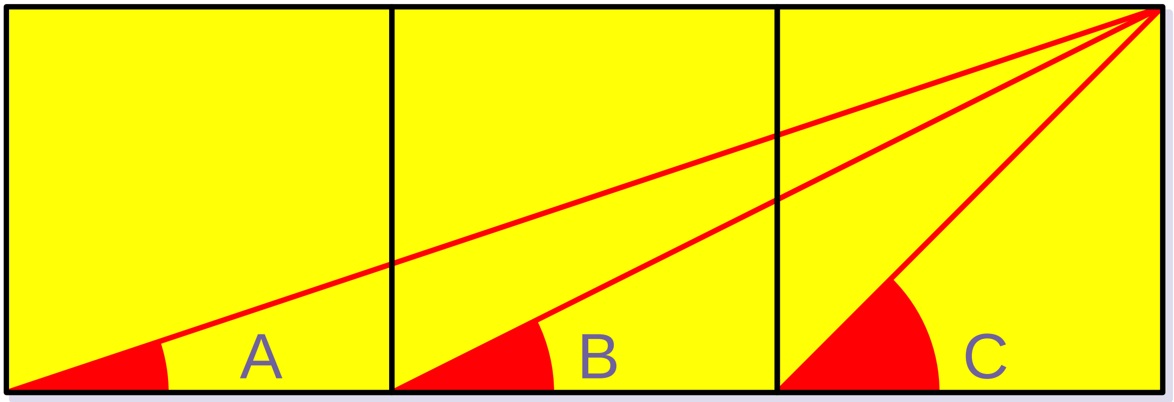
Hexagon Inside the Cube

after Martin Gardner

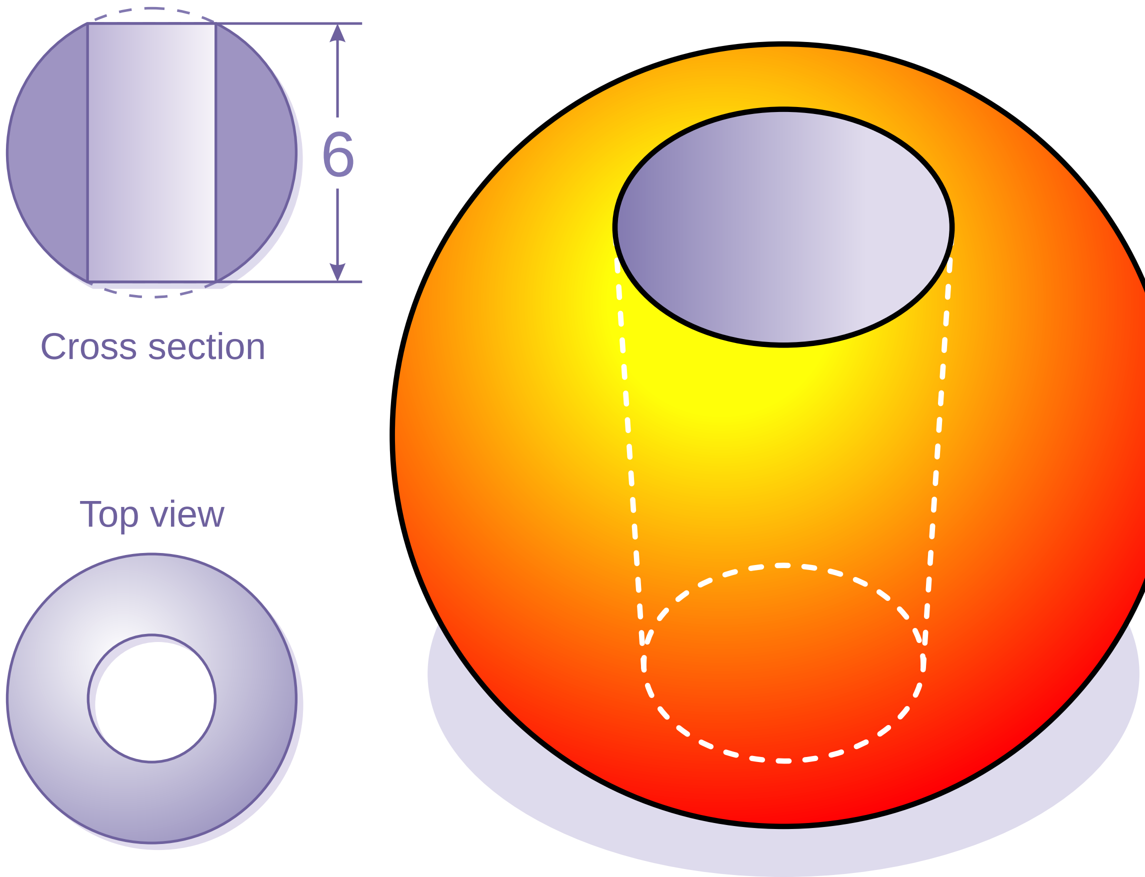


A plane which passes through the cube's center produces a cross section in form of a square (the leftmost cube in the illustration). A plane which passes through the three corners of the cube only produces a cross section in form of a regular triangle (the rightmost cube in the illustration).

The objective is to find the way how the plane should pass through the cube in order to produce a cross section that is a regular hexagon. If the cube's side is one unit, what is the side of the hexagon?



Using only elementary geometry (not even trigonometry), prove that angle C in the illustration equals the sum of angles A and B.



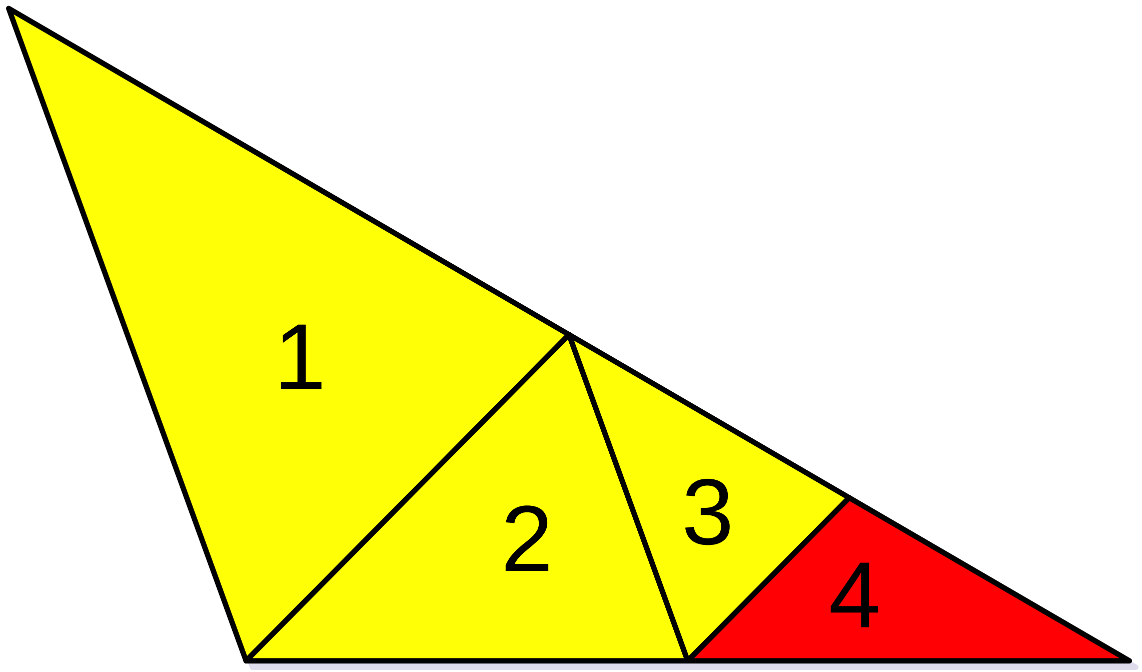
A cylindrical hole six inches long has been drilled straight through the center of a solid sphere - just as shown in the illustration.

What is the volume remaining in the sphere?

*Martin Gardner has found the earliest reference for this problem in Samuel I. Jones's Mathematical Nuts, self-published, Nashville, 1932.

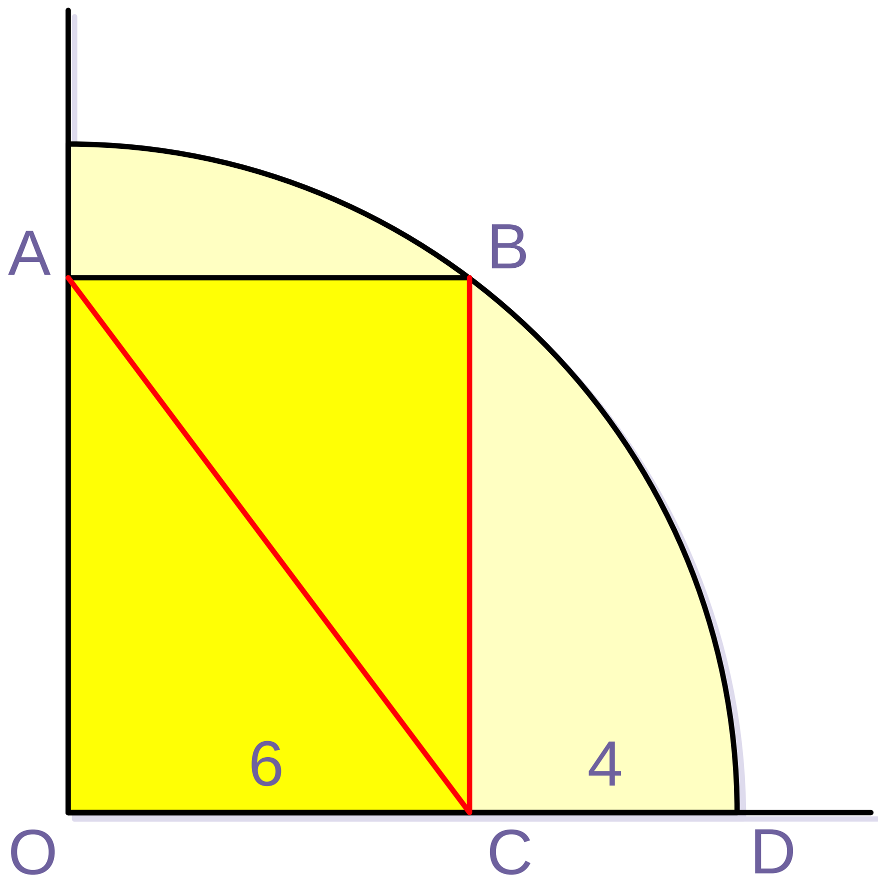
Acute Dissection

by Mel Stover



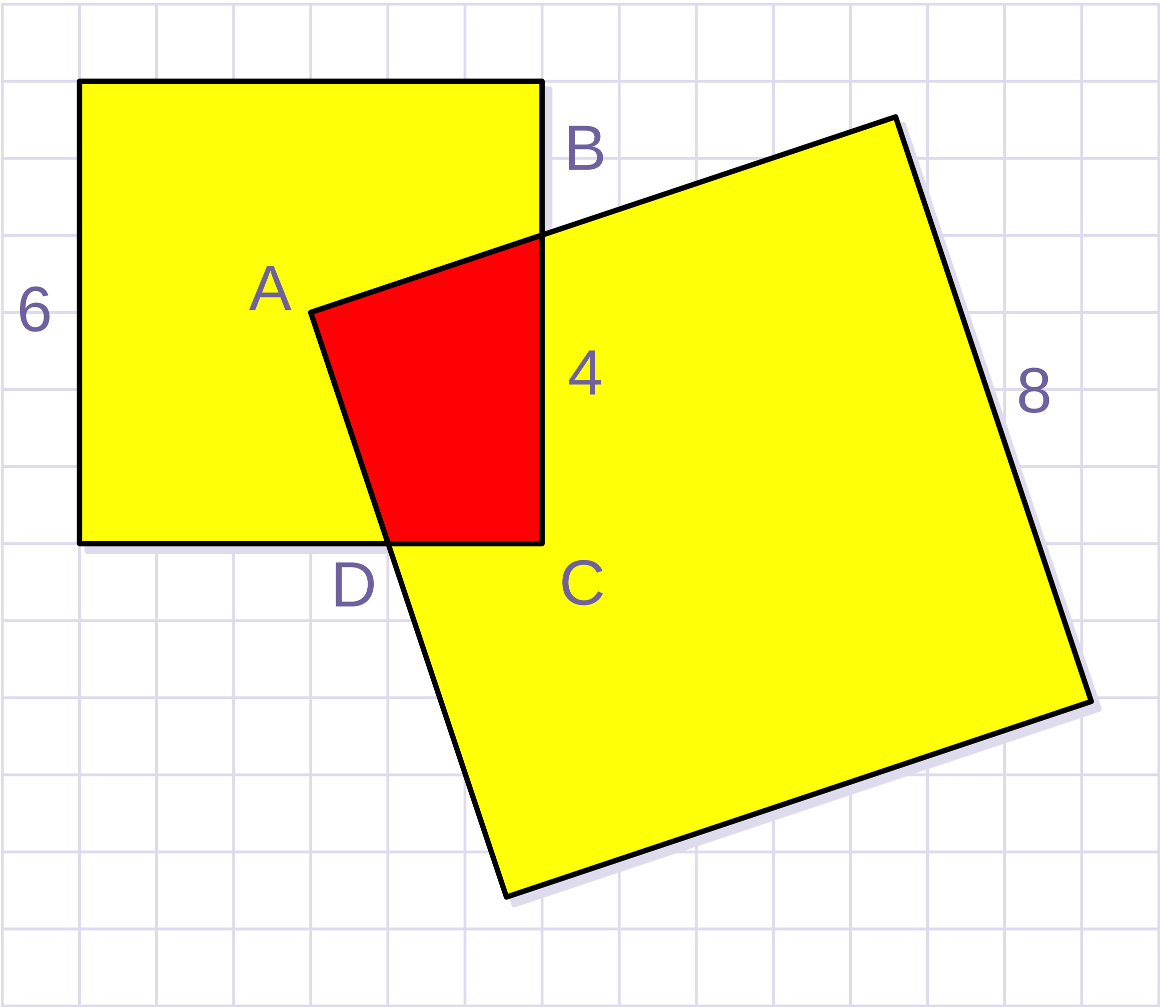
Is it possible to cut an obtuse triangle (a triangle with one obtuse angle) into smaller triangles, all of them acute? An acute triangle is a triangle with three acute angles. A right angle is neither acute nor obtuse. If such a dissection can be done, what is the smallest number of acute triangles into which any obtuse triangle can be dissected?

The illustration shows how an obtuse triangle can be divided into almost all acute triangles except one - the red one. Thus what approach should be used when it is required to cut an obtuse triangle into acute triangles only?



A rectangle is inscribed in the quadrant of a circle as shown in the illustration. Given some distances indicated, can you accurately determine the length of the diagonal AC?

A little extra question: can you also determine the length of the rectangle's side BC?

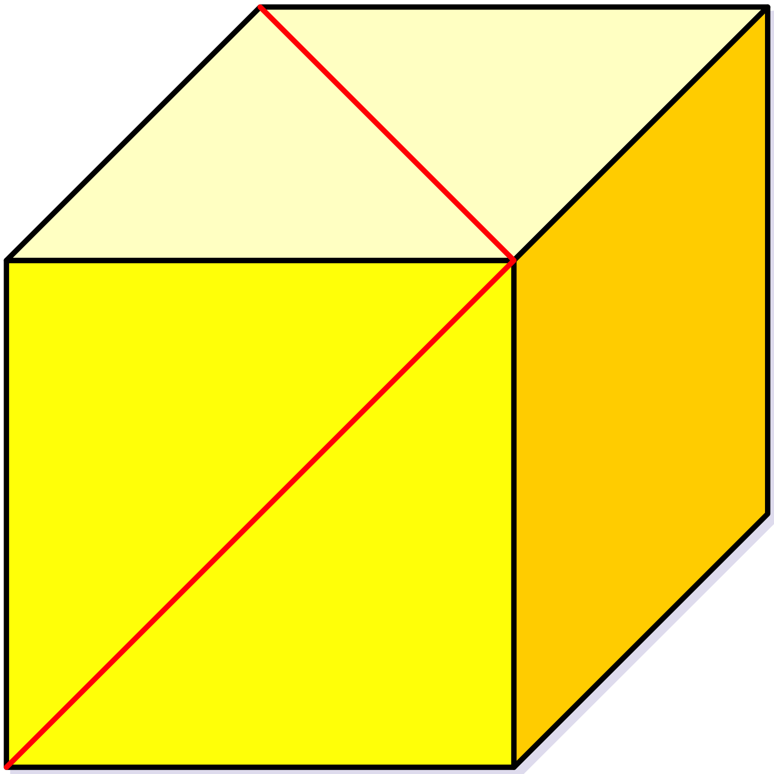


A square with side of eight units overlaps a square with side of six units in such a way that its corner A is placed exactly at the center of the small square. As the result of the overlapping the two sides of the large square intersect the two sides of the small square exactly at the points B and D as shown in the illustration. The BC line is 4 units long.

What is the area of overlap of the two squares, i.e. the area of the red quadrangle ABCD?

Cube Angle

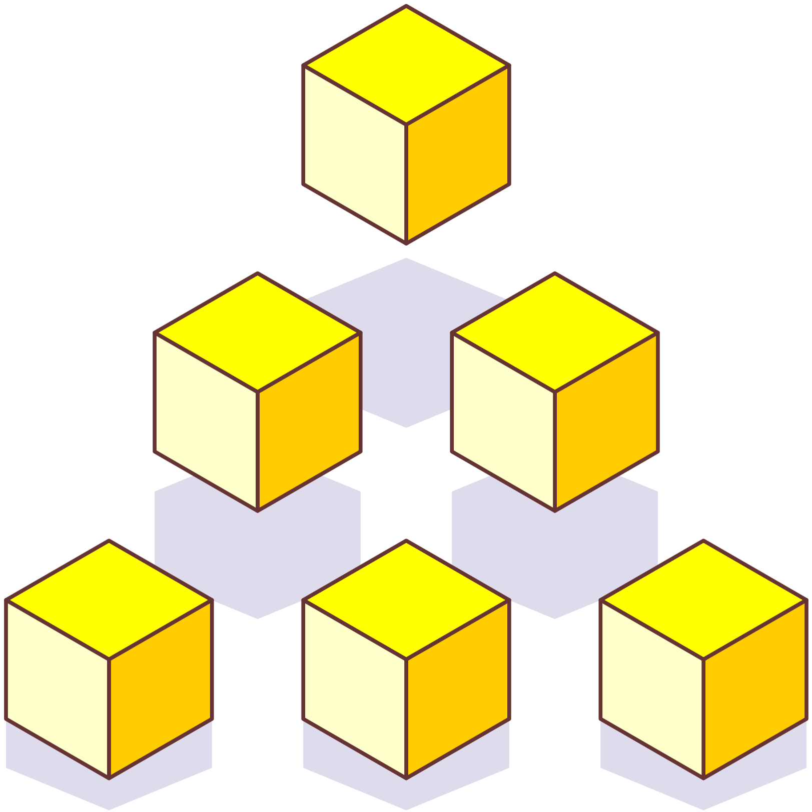
after Martin Gardner



Can you say what angle is made by the two red lines drawn on the two sides of the cube as shown in the illustration?

Six Cubes

by Martin Gardner



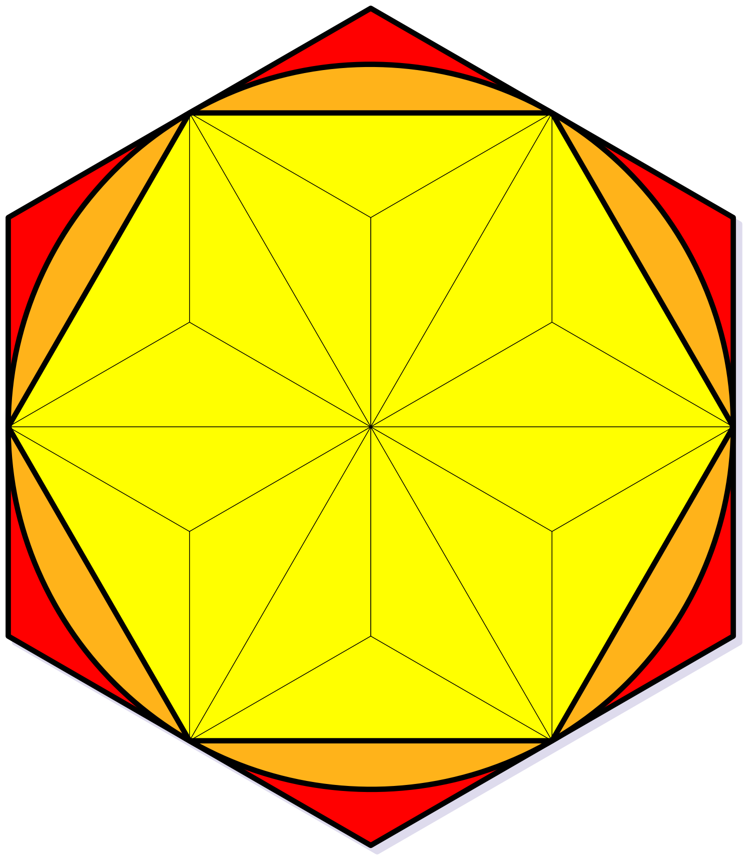
Take six identical cubes and place them on the table in such a way that each cube will touch every of the rest five cubes with some part of its side (touching along edges or at corners doesn't count).

You may use six identical matchboxes instead of cubes.

Solutions

Hexagon-Circle-Hexagon

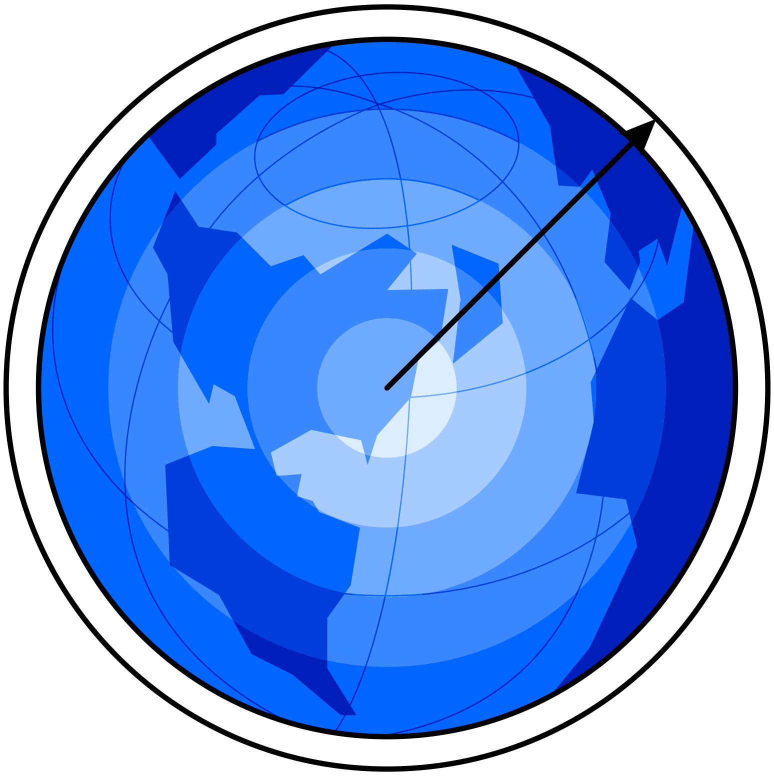
(Solution)



To solve the puzzle it is enough to turn the smaller hexagon as shown in the illustration. The inner straight lines divide the larger hexagon into 24 congruent triangles, 18 of which form the smaller hexagon. The ratio of areas is 18 : 24 = 3 : 4, and so if the smaller hexagon has an area of three, the larger one has as area of four.

The Earth Belt

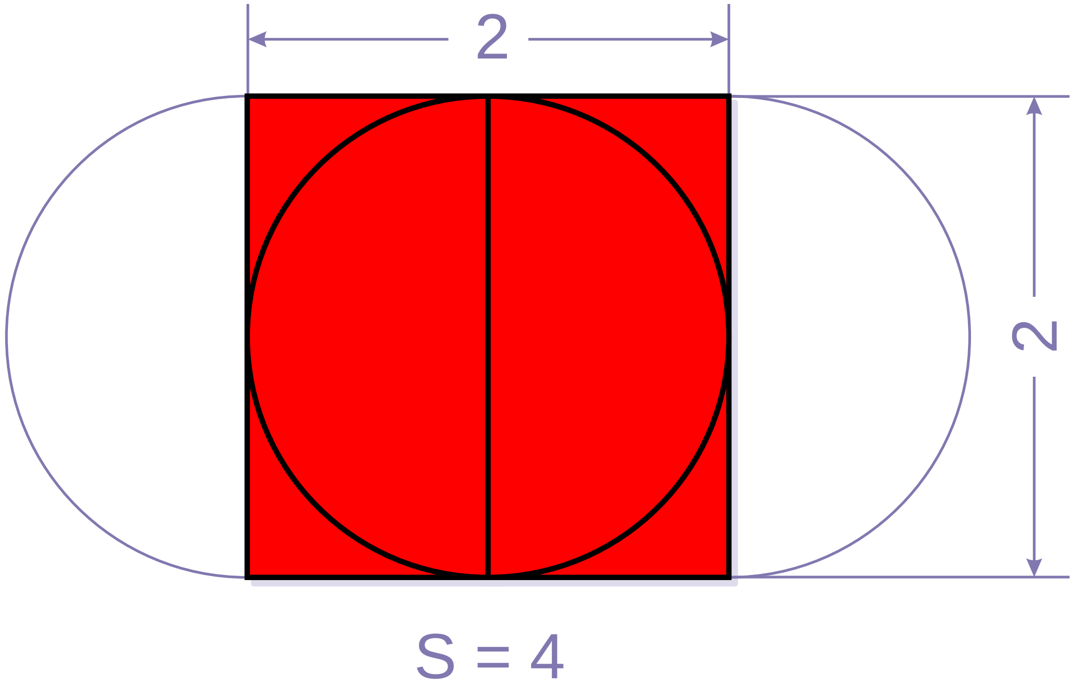
(Solution)



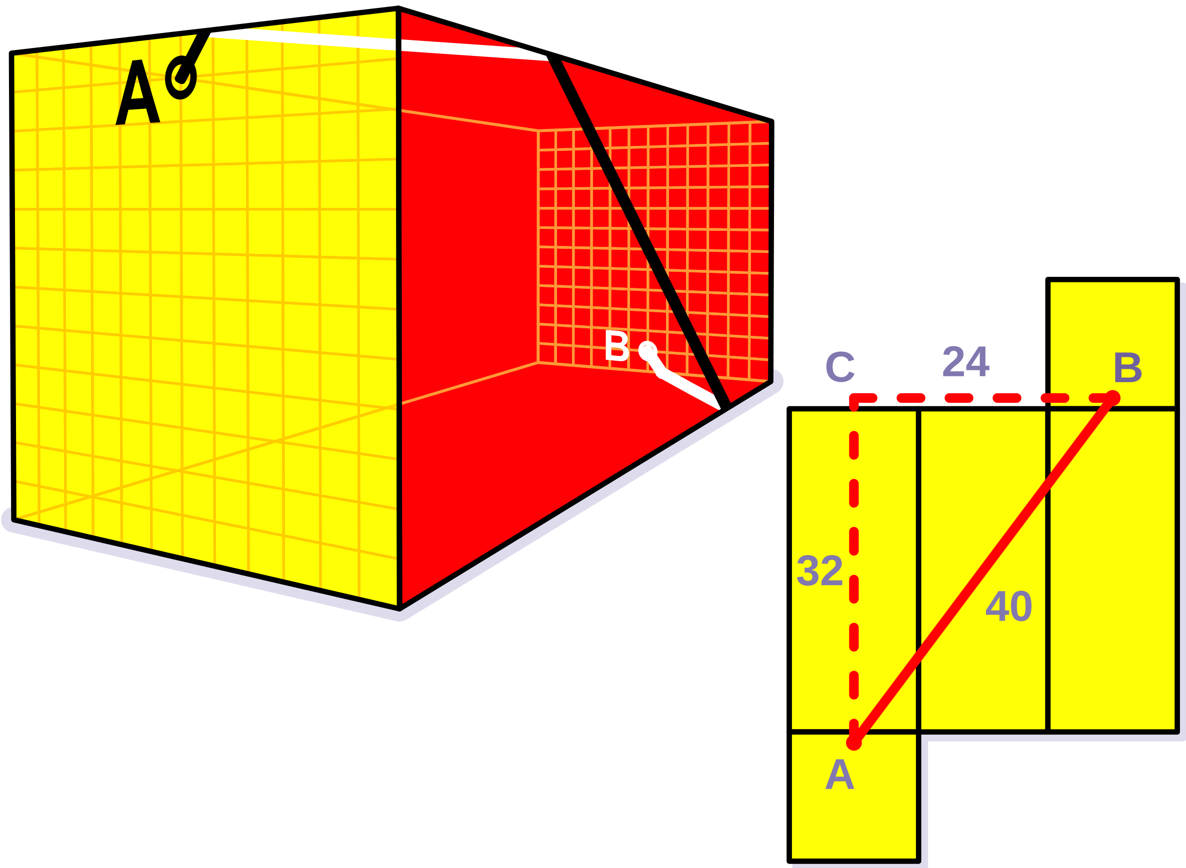
It seems surprising, but that steel belt, after a meter is added to it (approximately a yard and four inches), will be raised 15+ centimeters (approximately six inches) all the way around! This is certainly high enough for a baseball to pass underneath.

Actually, the height the belt is raised on, is the same regardless of how large the sphere is. It is easy to see why. When the belt is tight around the sphere, it makes the circumference of a circle with a radius that is the same as the radius of the sphere. As it is known from plane geometry the circumference of a circle is equal to its diameter (which is twice its radius) times pi. Pi is 3.14+. Therefore, if the circumference of any circle is increased by one meter, the diameter of the circle is increased by a trifle less than one-third of a meter, or 31+ centimeters (a trifle more than a foot). This means, of course, that the radius will increase by almost 15+ centimeters (approximately six inches).

As it is shown in the illustration, this increase in radius is the height that the belt will be raised from the sphere's surface. It will be exactly the same, 15+ centimeters (almost six inches), regardless of whether the sphere be the size of the sun, of the earth or of an orange!



The two semicircles together form a circle that fits the hole. The remaining paper therefore has a total area of four square units.

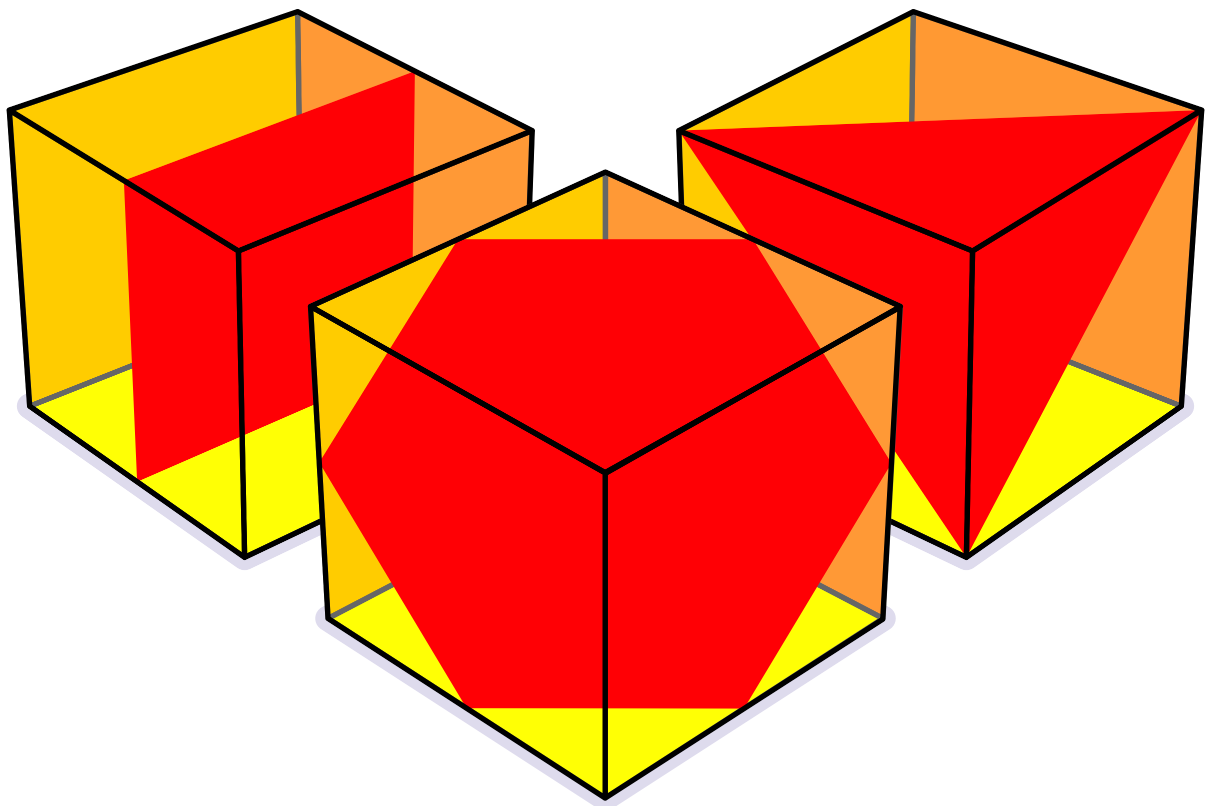


At first glance it seems that the shortest distance between A and B is the straight way along the sides of the box, i.e. 1 unit up, 30 units along the top side of the box and then 11 units down the opposite side - 42 units in total. But as it was stated in the hint the shortest distance between A and B is less than 42.

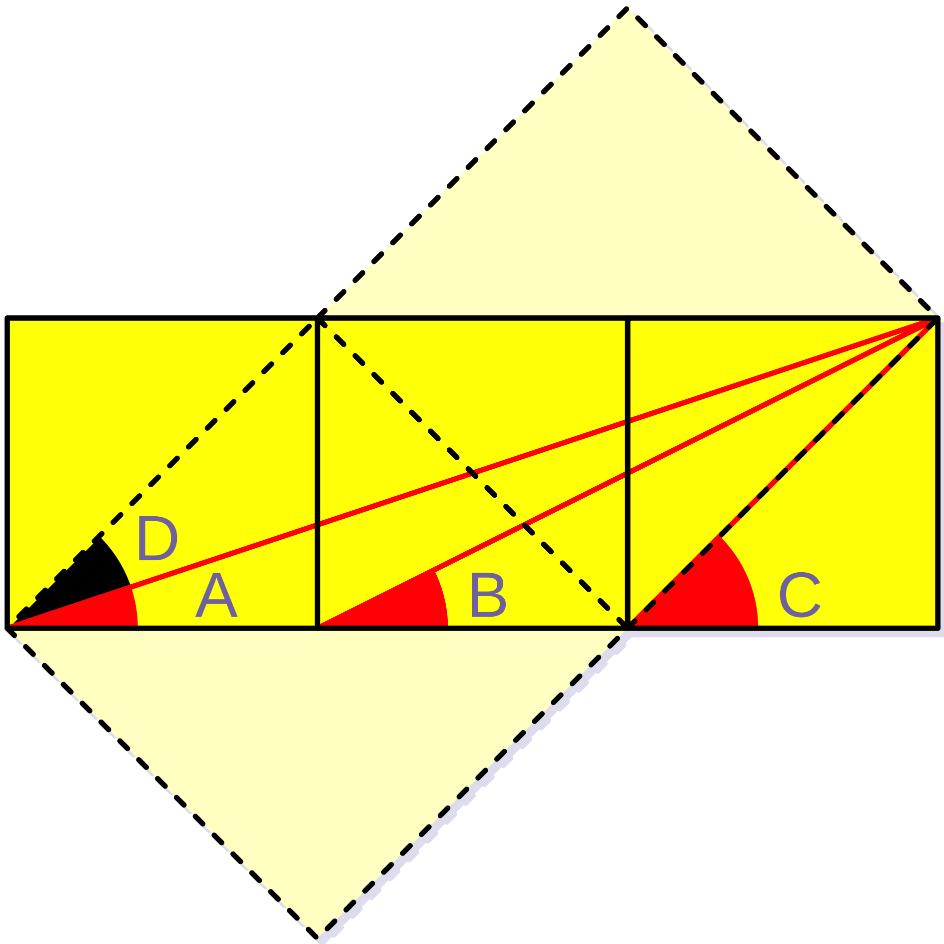
To find the shortest distance out it is useful to unfold some sides of the box into a 2D model. If to unfold them as shown in the lower right corner of the illustration, we get a right triangle where the hypotenuse AB is the distance between the two spots. It equals square root of (AC^2 + BC^2). AC is 32 units long (1 + 30 + 1) and BC is 24 units long (6 + 12 + 6). Thus the distance AB equals square root ($32^2 + 24^2$) = square root of 1600 or 40.

Hexagon Inside the Cube

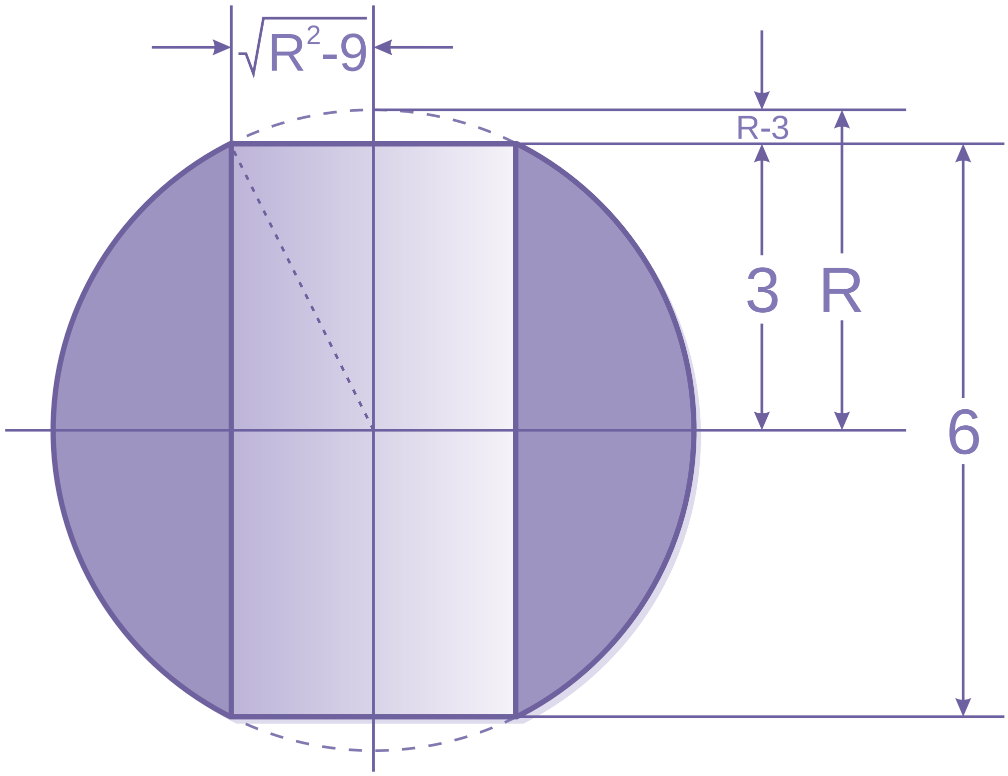
(Solution)



A plane which passes through the midpoints of six sides of the cube as shown in the center of the illustration, produces a cross section that is a regular hexagon. Since the side of the cube is one unit, the side of the hexagon is square root of ½.



Let's construct the additional squares indicated by dotted lines. It is clear from the illustration that angle C is the sum of angles A and D. Angle B equals angle D because they are corresponding angles of similar right triangles (with the respective legs in the 1:2 proportion). That means B can be substituted for D, which automatically makes the C equals the sum of A and B.



If you want to avoid the calculations in Solution 1, simply take a look directly at Solution 2 below it.

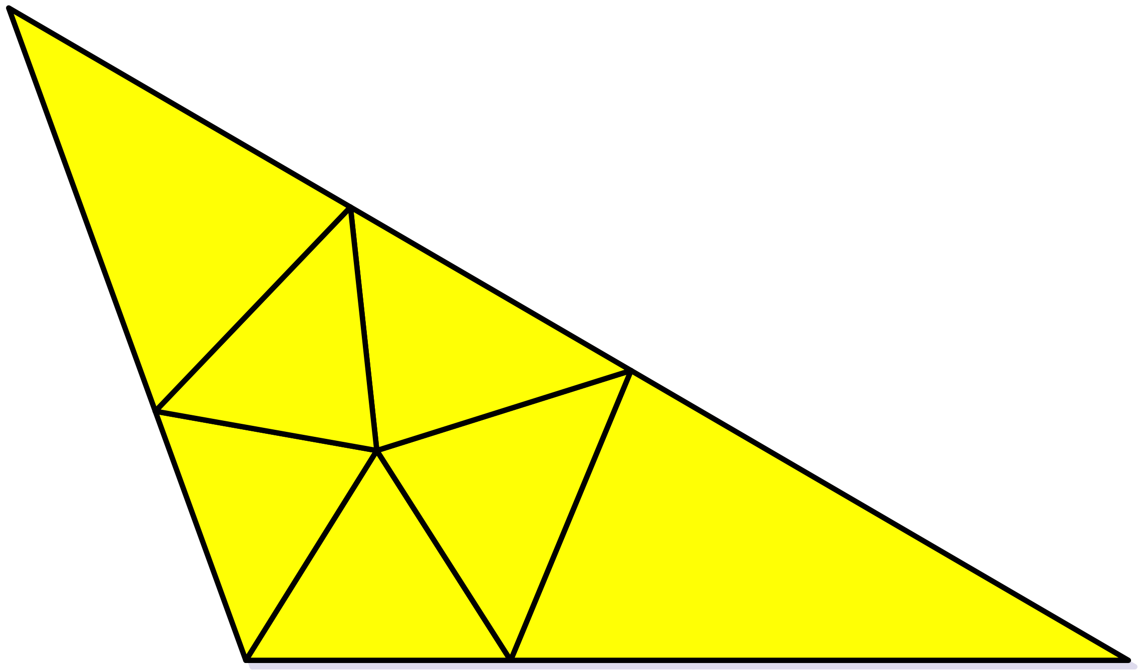
Solution 1. Let **R** be the radius of the sphere. As the illustration indicates, the radius of the cylindrical hole will then be the square root of $\mathbf{R^2 - 9}$, and the altitude of the spherical caps at each end of the cylinder will be **R - 3**. To determine the residue after the cylinder and caps have been removed, we add the volume of the cylinder, $\mathbf{6\pi(R^2 - 9)}$, to twice the volume of the spherical cap, and subtract the total from the volume of the sphere, $\mathbf{4\pi R^3/3}$. The volume of the cap is obtained by the following formula, in which **A** stands for its altitude and **r** for its radius: $\mathbf{\pi A(3r^2 + A^2)/6}$.

When this computation is made, all terms obligingly cancel out except **36π** - the volume of the residue in cubic inches. In other words, the residue is constant regardless of the hole's diameter or the size of the sphere!

Solution 2. John W. Campbell, Jr., editor of *Astounding Science Fiction*, was one of several readers who solved the sphere problem quickly by reasoning adroitly as follows: The problem would not be given unless it has a unique solution. If it has a unique solution, the volume must be a constant which would hold even when the hole is reduced to zero radius. Therefore the residue must equal the volume of a sphere with a diameter of six inches, namely **36π**.

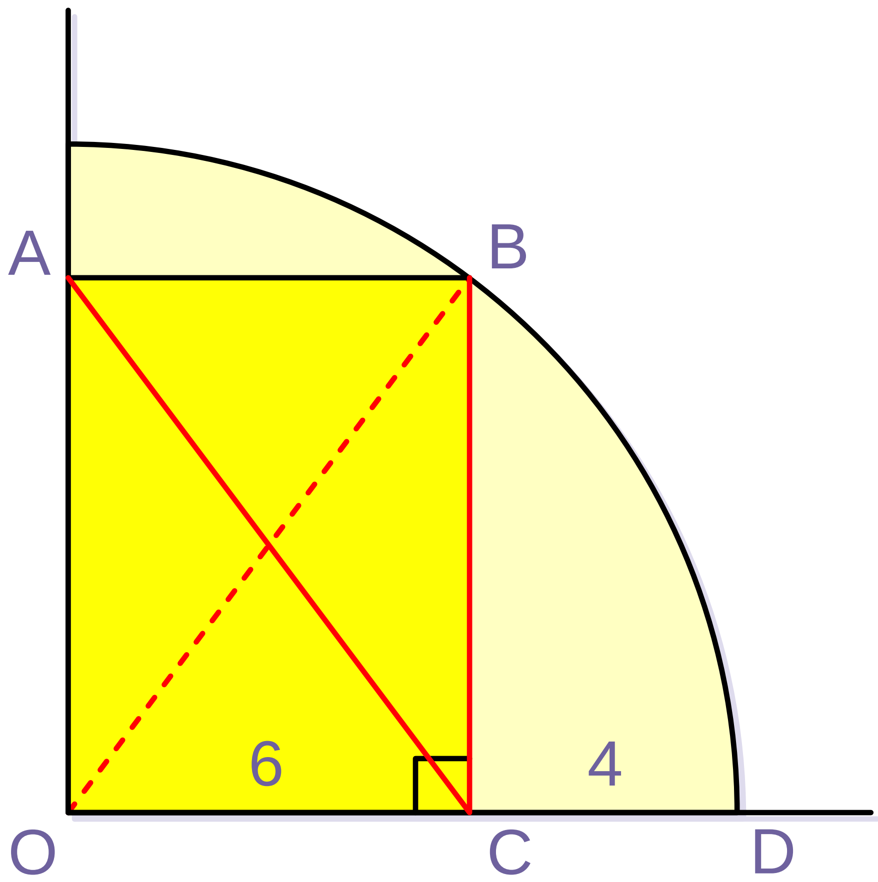
Acute Dissection

(Solution)



The minimal number of acute triangles is 7. The pattern for the seven triangles is shown in the illustration. An elegant proof for the seven acute triangles has been provided by Wallace Manheimer in American Mathematical Monthly, November 1960. The logic behind the proof is as follows.

The obtuse angle must be divided by a line. This line cannot go all the way to the other side, for then it would form another obtuse triangle (or two triangles with right angles), which in turn would have to be dissected, consequently the pattern for the large triangle would not be minimal. The line dividing the obtuse angle must, therefore, terminate at a point inside the triangle. At this vertex, at least five lines must meet, otherwise the angles at this vertex would not all be acute. This creates the inner pentagon of five triangles, making a total of seven triangles as shown in the illustration.

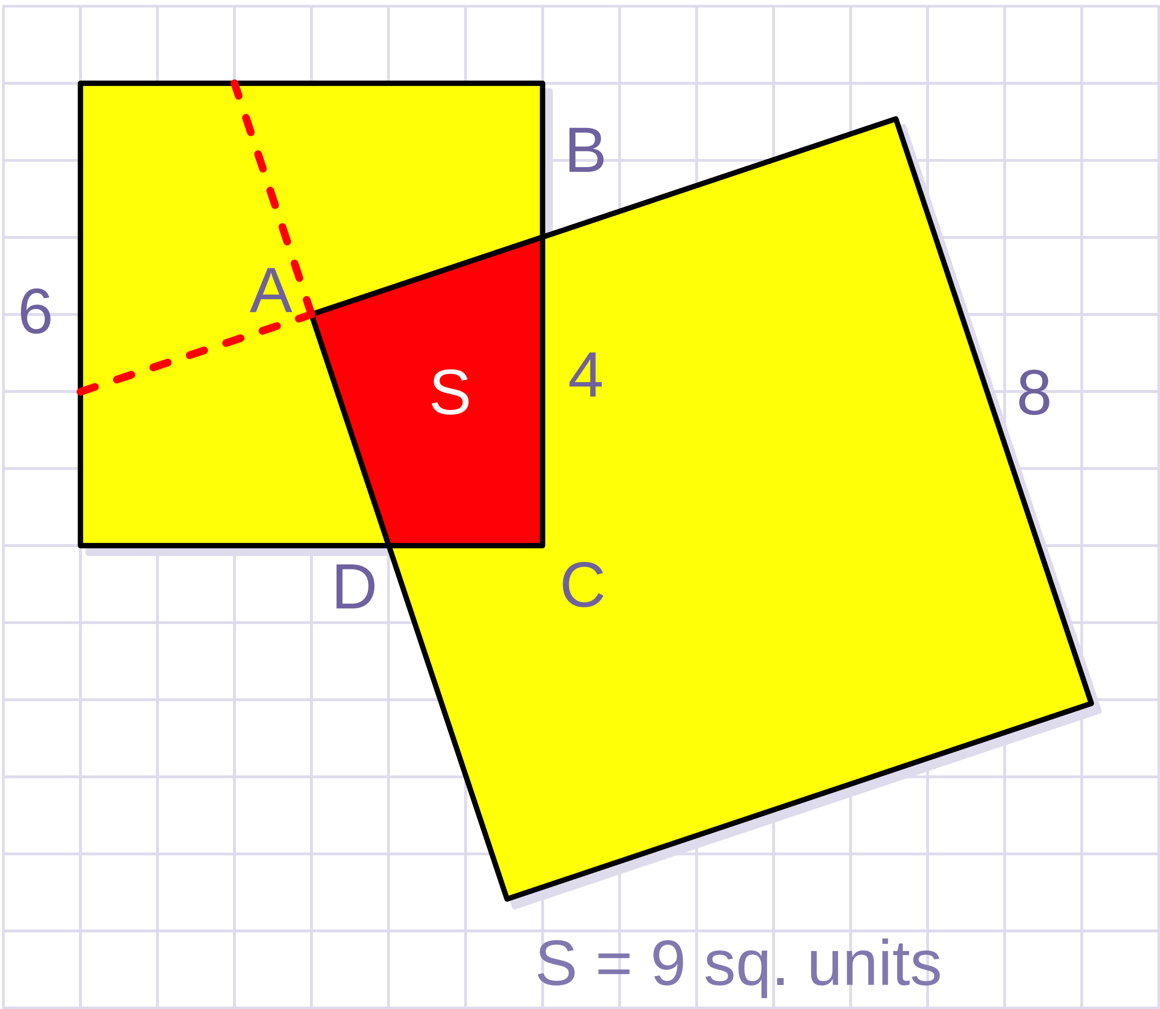


The line AC is one diagonal of the rectangle. As it can be seen form the illustration the other its diagonal BO is exactly equal to the radius of the circle, i.e. OC plus CD, 6 plus 4, or 10 units. Since the diagonals of a rectangle are equal, AC is 10 units long too.

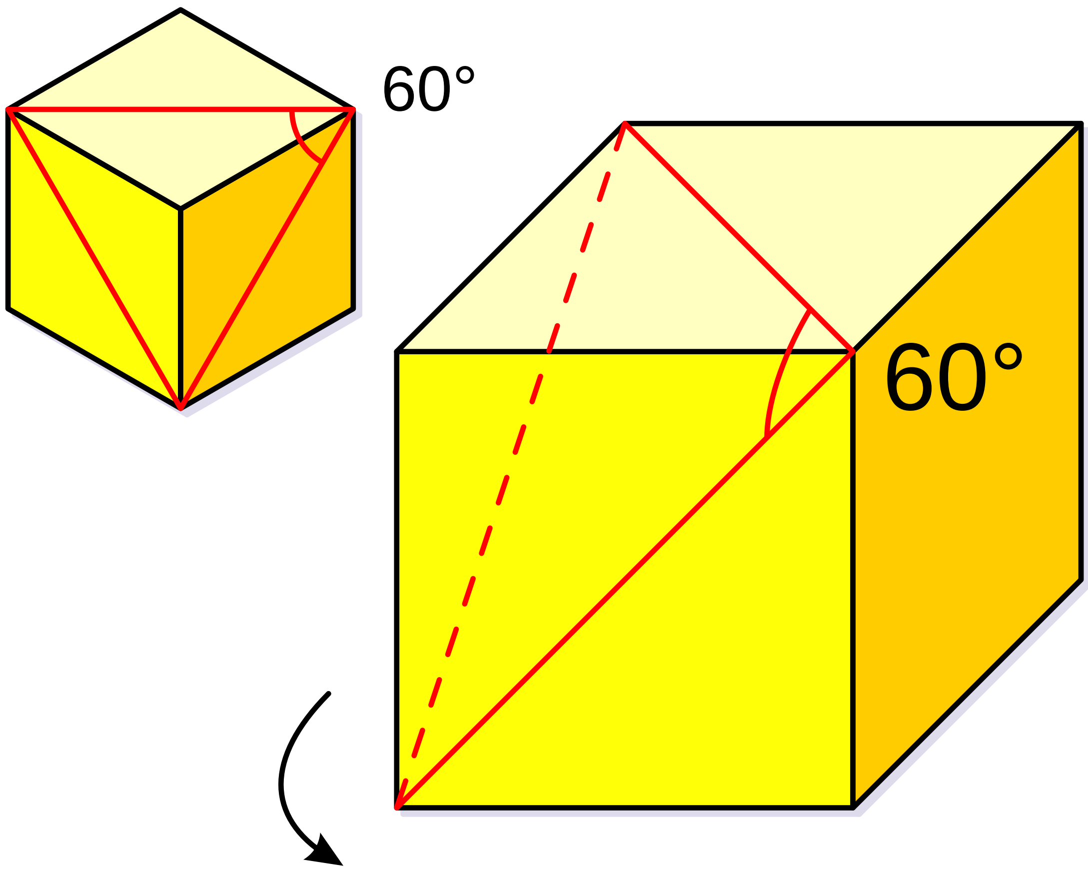
Using the Pythagorean Theorem it can be figured out that the side BC is 8 units long ($10^2 - 6^2 = 8^2$).

Square Intersection

(Solution)



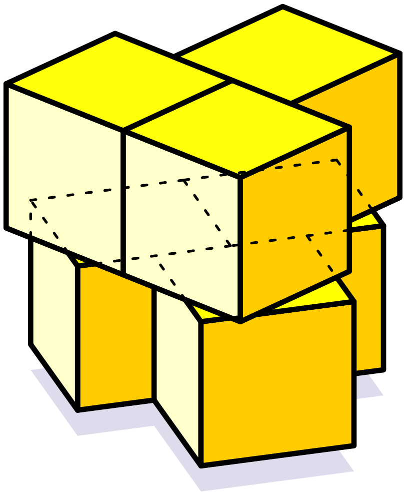
To solve this puzzle just extend two sides of the large square as shown by the dotted lines in the illustration. This obviously divides the small square into four congruent parts. Since the small square has an area of 36 square units (6 x 6), the overlap (red quadrangle) must have an area of 36/4, or 9 square units. The amusing thing about the problem is that the area of overlap is constant regardless of the large square's position as it rotates around A. The fact that BC is 4 units long is actually irrelevant information.

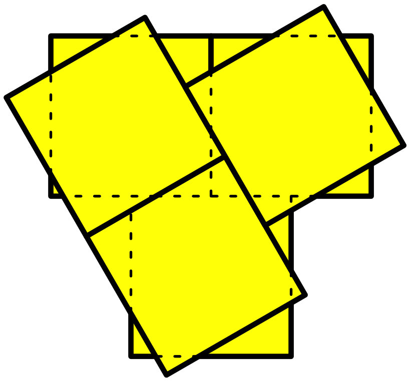


The angle made by the two lines is 60 degrees. When we join the ends of the two lines with another line we get a triangle which has three its sides equal to the diagonal of the cube's side as shown in the illustration. Thus we get an equilateral triangle. As far as each angle in an equilateral triangle is equal to 60 degrees, it means the angle between the two lines is also 60 degrees.

Six Cubes

(Solution)





top view

The solution is shown above.